Case Study Using Allocation Method 3: Option Pricing Method (OPM)

As discussed earlier, breakpoints are inflection points in equity value at which those equity holders who have contractually defined choices would optimally decide to change their participation in liquidation proceeds. Therefore, the breakpoints set apart specific regions across the lognormal distribution and in the context of the OPM are equivalent to different call option strike prices on the value of the firm.

Lognormal Distribution

In this case study it is assumed that future equity values follow a lognormal distribution. This is a common assumption used in the expected path of future stock returns and the same assumption used in the Black–Scholes option pricing model.

The lognormal distribution takes its name from the fact that it represents random outcomes whose natural logarithm follows the normal distribution. Unlike the normal distribution, the lognormal distribution is bounded on the left by zero; but it is unbounded on the right. This makes it useful for situations where values (in this case equity value in Exhibit 30–7) are positively skewed and cannot be negative. If the logarithm of a random outcome is normally distributed, then the outcomes are lognormally distributed.

In the Black–Scholes option pricing model, stock returns are expressed as geometric rates of return (i.e., percentage rates of change and differences in logarithms are percentage changes) and those returns are normally distributed.

Assumptions of Case Study

The general assumptions of the case study as presented in the CVM section, plus new assumptions based on an assumed five-year time to liquidity, are as follows:

General:

> Equity Value = $1,800,000
> Equity Volatility = 50.0%
> Term = 5.0 years
> Risk-Free Interest Rate = 1.0%

Also recall the three breakpoints are $1,500,000, $1,930,000, and $2,557,000 from the CVM example. The same breakpoints can be employed to isolate the value of the four regions that are relevant to the analysis:

Region 1: Equity value ranging between $0 and $1,500,000
Region 2: Equity value ranging between $1,500,000 and $1,930,000
Region 3: Equity value ranging between $1,930,000 and $2,557,000
Region 4: Equity value exceeding $2,557,000

One could isolate the value of each region by modeling a series of call options as shown in Exhibit A30–6.

Exhibit A30–6

### Equity and Option Values as Value TEV (MVIC) Increases

	Call 0	Call 1	Call 2	Call 3	
Equity Value	$1,800,000	$1,800,000	$1,800,000	$1,800,000	
Strike Price	$0	$1,500,000	$1,930,000	$2,557,000	
Equity Volatility	50.0%	50.0%	50.0%	50.0%	
Risk-Free Interest Rate	1.0%	1.0%	1.0%	1.0%	
Expected Term	5.0 years	5.0 years	5.0 years	5.0 years	
Option Value	$1,800,000	$883,594	$752,704	$609,602	
TEV Region	1	2	3	4	**Total**
TEV Range ($000)	[$0 – $1,500]	[$1,500 – $1,930]	[$1,930 – $2,557]	$2,557 +	
Call n minus Call n+1	$916,406	$130,890	$143,102	$609,602	$1,800,000
% Allocation	50.9%	7.3%	8.0%	33.9%	100.0%

Note the strike prices of the options are set equal to the breakpoints and the equity value to the current equity value estimate of $1,800,000. The options are modeled over a period of five years on the assumption the assets would be sold in year five. The OPM involves modeling multiple options using the Black–Scholes option pricing model. Therefore, the appropriate drift rate for the model is the term-matched risk-free interest rate of 1%. The interaction between the expected term and the equity volatility of 50% determines the range of future outcomes.

As discussed before, the value of each region can be calculated as either the value of a call option, for the last region, or as the difference between the values of two call options. In this example, the four regions can be represented as shown in Exhibit A30–7.

**Values of Preferred Stock, Common Stock and Options
as Value of Total Equity Increases**

Call Option Payoff
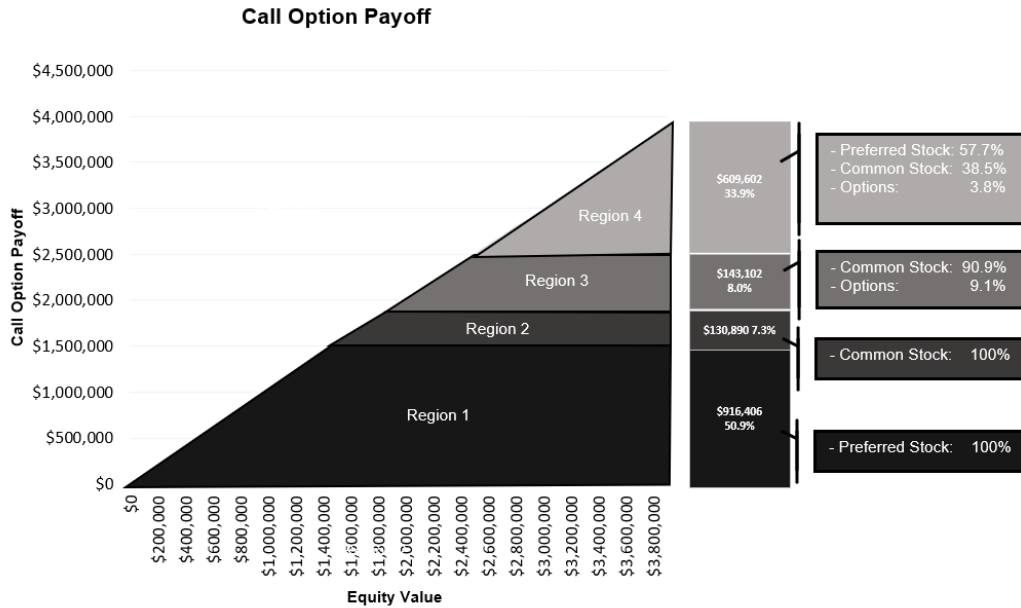
For example, the value of the first region of $916,406 associated with equity value outcomes ranging between $0 and $1,500,000 can be calculated as the difference between the value of Call 0 of $1,800,000 and that of Call 1 of $883,594. Similarly, the value of the second region of $130,890 associated with outcomes ranging between $1,500,000 and $1,930,000 can be calculated as the difference between the value of Call 1 of $883,594 and that of Call 2 of $752,704. Note the value of the last region where the equity value exceeds $2,557,000 is simply worth $609,602, the value of the last option. The regions have different, and far less intuitive, values than the PWERM example in part because the PWERM uses unweighted future values whereas here the values are probability weighted as well as discounted to present value.

As reported in the last row of Exhibit A30–6, the OPM allocates 50.9% of the current equity value to the first region, 7.3% to the second region, 8.0% to the third region and 33.9% to the last region. Once the values of the regions are isolated, the next step involves determining what percentage of these values should be allocated to each security. To address this question, it helps to write down the number of common shares that apply in each region as presented in Exhibit A30–8.

Exhibit A30–8

Allocation of Values of Preferred Stock, Common Stock and Options as Value of Total Equity Increases

Equity Value	Description	Common Shares	Percentage Allocated
$0 to $1,500,000	Preferred receive liquidation preference.	N/A because the equity	Common 0%
	Options are not exercised.	value is worth less than	Options 0%
	Preferred do not convert.	the liquidation preference.	Preferred 100%
	Common and options receive nothing.		Total 100%
$1,500,000 to $1,930,000	Options are not exercised.	Common 1,000,000	Common 100%
	Preferred do not convert.	Options 0	Options 0%
	Common receive 100% of the value.	Preferred 0	Preferred 0%
		Total 1,000,000	Total 100%
$1,930,000 to $2,557,000	Options are exercised.	Common 1,000,000	Common 91%
	Preferred do not convert.	Options 100,000	Options 9%
	Common and options share the value on a pro-rata basis.	Preferred 0	Preferred 0%
		Total 1,100,000	Total 100%
$2,557,000+	Options are exercised.	Common 1,000,000	Common 38%
	Preferred convert into common shares.	Options 100,000	Options 4%
	Common, options and preferred share the value on a pro-rata basis.	Preferred 1,500,000	Preferred 58%
		Total 2,600,000	Total 100%

The percentages reported in the last column represent the percentage of the regions' values each security receives. For example, when the equity value falls below $1,500,000, the preferred stockholders receive their liquidation preference. The options and common shares receive nothing because there is no residual left to be allocated. When the equity value ranges between $1,500,000 and $1,930,000, the options are not exercised and the preferred stock does not convert. Therefore, in the second region all the value is allocated to the common stock. When the equity value ranges between $1,930,000 and $2,557,000, the options are exercised and the preferred stockholders do not convert. The 100,000 options are added to the common stock shares to bring the total number of shares to 1,100,000. The options share the value of the third region on a pro-rata basis with common stock, net of exercise proceeds. Finally, when the equity value exceeds $2,557,000, the preferred stock converts into common stock. The 1,500,000 shares of preferred converted 1-to-1 into common shares are added to the total to bring the total number of shares to 2,600,000. The preferred stockholders share the value of the fourth region on a pro-rata basis with the other shares.

The information reported in the last two columns can be added to the analysis to bring everything together. Exhibit A30–9 illustrates the allocation of the current equity value in the OPM with all its components.

Exhibit A30–9

Allocation of Equity Value using OPM

	Call 0	Call 1	Call 2	Call 3	
Equity Value	$1,800,000	$1,800,000	$1,800,000	$1,800,000	
Strike Price	$0	$1,500,000	$1,930,000	$2,557,000	
Equity Volatility	50.0%	50.0%	50.0%	50.0%	
Risk-Free Rate	1.0%	1.0%	1.0%	1.0%	
Expected Term	5.0 yrs	5.0 yrs	5.0 yrs	5.0 yrs	
Option Value	$1,800,000	$883,594	$752,704	$609,602	

TEV Region	1	2	3	4	Total
TEV Range ($000)	[$0 – $1,500]	[$1,500 – $1,930]	[$1,930 – $2,557]	>$2,557	
Call $_n$ minus Call $_{n+1}$	$916,406	$130,890	$143,102	$609,602	$1,800,000
% Allocation	50.9%	7.3%	8.0%	33.9%	100.0%

Shares					
Preferred Stock	1,500,000	0	0	1,500,000	
Common Stock	0	1,000,000	1,000,000	1,000,000	
Options	0	0	100,000	100,000	
	1,500,000	1,000,000	1,100,000	2,600,000	

Percentage Allocation					
Preferred Stock	100.0%	0.0%	0.0%	57.7%	
Common Stock	0.0%	100.0%	90.9%	38.5%	
Options	0.0%	0.0%	9.1%	3.8%	
	100.0%	100.0%	90.9%	38.5%	

Allocation of Value					Total
Preferred Stock	$916,406	$0	$0	$351,693	$1,268,099
Common Stock	$0	$130,890	$130,093	$234,462	$495,445
Options	$0	$0	$13,009	$23,446	$36,455
	$916,406	$130,890	$143,102	$609,602	$1,800,000

At the top, the call options with strike prices matching the breakpoints are valued using Black–Scholes option pricing model (see Exhibit A30–6). In the middle, each region is assigned a value by taking the difference between two call options. Finally, at the bottom the value of each region is allocated across the company's securities based on their respective pro-rata percentages.

More specifically, the values of the different regions of $916,406, $130,890, $143,102, and $609,602 respectively are allocated across the securities based on the securities' corresponding pro-rata percentages reported in the second to last section of Exhibit A30–9 (labeled "percentage allocation"). For example, the preferred stockholders receive 100% of the value of the first region. Therefore, the value of the first region that gets allocated to the preferred stock at the bottom of Exhibit A30–9 can be calculated as the product of 100% times the value of the first region of $916,406. As another example, the value of the third region of $143,102 gets allocated between common stock and options assuming a 90.9% and 9.1% split between them to get a value of $130,093 for common stock and

$13,009 for options. In the OPM, the option proceeds do not enter the calculations because the options are assumed to net settle. In other words, when the options are exercised there is no additional inflow of money to the company and the options and common stockholders share the value of the third region on a pro-rata basis.

After the value of each region is allocated to the securities, the value of the securities can be calculated by summing up the amounts reported along each row. In this case, the OPM produced a value of $1,268,099 for the preferred stock, $495,445 for the common stock, and $36,455 for options. Note when the OPM analysis is performed correctly the sum of the values of the securities reported in the bottom right corner of Exbibit A30–9 reconciles to the current equity value estimate of $1,800,000. In other words, the OPM, like the CVM or PWERM, allocates the current equity value between the securities. No value gets created or destroyed in the process.