USING PUT OPTION MODELS TO DETERMINE A REASONABLE DLOM

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INTRODUCTION

- The discount for lack of marketability (DLOM) can be modeled as the value of a foregone put option.
- The holder of an asset with impaired liquidity, for example, due to marketability restrictions, has a reduced ability to resell the asset or transfer it.
- This loss of flexibility can be thought of as the loss of a put option.
- The cost of the loss of flexibility is the value of this foregone put option.

OVERVIEW OF PRESENTATION

- Distinguish marketability discount from a liquidity discount.
- Compare three put option models that are used to calculate the DLOM.
- Provide sample calculations to demonstrate the reasonableness of the model discounts.
- Work two examples to illustrate the application of the DLOM put option models.

MARKETABILITY DISCOUNT VERSUS LIQUIDITY DISCOUNT

- It is useful to distinguish lack of liquidity from lack of marketability.
- Marketability refers to the asset holder's legal and contractual ability to sell or otherwise transfer ownership of the asset.
- Liquidity also refers to an asset holder's flexibility to transfer asset ownership.
- Liquidity is the relative ease with which the asset holder can convert the asset into cash without sacrificing any portion of the asset's intrinsic value.

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MARKETABILITY DISCOUNT VERSUS LIQUIDITY DISCOUNT (cont'd)

- A liquid market gives an asset holder the flexibility to sell an asset at any time of her choosing without any loss of intrinsic value.
- Lack of liquidity entails a loss of timing flexibility because the asset holder can not dispose of the asset quickly unless he or she is willing to accept a discount.
- An asset that lacks marketability also lacks liquidity but the lack of liquidity is due to legal or contractual restrictions.
- The put option models are specifically designed to measure the discount for lack of marketability during a specified period of duration *T*.
- They can be extended to measure liquidity discounts.

MARKETABILITY DISCOUNT VS. PRIVATE PLACEMENT DISCOUNT

- Marketability discount is different from the private placement discount.
- Private placement discounts reflect much more than just transfer restrictions: information/ownership effects, information gathering costs, and compensation for risk of negative post-issue performance.
- Marketability discounts should generally be less than private placement discounts.
- One can't simply use private placement discounts to determine marketability discounts unless adjustments are made for the effects that are unrelated to transfer restrictions.

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ARE PRIVATE PLACEMENT DISCOUNTS A RELIABLE INDICATOR OF THE MARKETABILITY DISCOUNT?

- The stock market generally reacts favorably to the announcement of a common stock private placement.
- The announcement conveys valuable information.
- The issuance of common stock involves equity ownership concentration effects, too.
- More than just lack of free marketability is responsible for the private placement discounts we observe.

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REASONS FOR THE PRIVATE PLACEMENT DISCOUNT

- Information and ownership structure effects/Compensation private placement investors require for monitoring services (Wruck, 1989).
- Illiquidity (Silber, 1991).
- Information gathering during the due diligence process (Hertzel and Smith, 1993).
- Negative post-issue stock-price performance/Discount reflects private investors' assessment of true (lower) firm value (Hertzel, Lemmon, Linck, and Rees, 2002).

IMPLICATION

- The observed private placement discounts should be adjusted for the information and equity ownership concentration effects in order to isolate the impact of the lack of marketability.

PUT OPTION DLOM MODELS

- Black-Scholes-Merton (BSM) standard put option model (Black and Scholes, 1973; Merton, 1973).
- Lookback put option model (Longstaff, 1995).
- Average-strike put option model (Finnerty, 2012).

BLACK-SCHOLES-MERTON (BSM) PUT OPTION DLOM MODEL

- BSM standard put option pricing model (OPM) formula is:

$$p = Ke^{-rT}N(-d_2) - S_0N(-d_1)$$

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

BSM DLOM MODEL ASSUMPTIONS

- The BSM put option OPM assumes a fixed strike price for the entire restriction period.
- It measures the loss of resale flexibility with respect to a constant share price (i.e., the initial share price when the put option is initially at-the-money).
- The shareholder is assumed to lose the ability to sell the shares at this price during the entire restriction period regardless of volatility.

BSM DLOM MODEL ASSUMPTIONS (cont'd)

- The length of the restriction period is fixed.
- The BSM put OPM is therefore a model for approximating marketability discounts when the restriction period is fixed (Chaffee, 1993).

MAIN LIMITATION OF BSM PUT OPM

- The BSM put OPM will not measure the DLOM accurately because it assumes the strike price is fixed.
- This assumption limits the BSM model's applicability to DLOMs because the potential selling prices the asset holder faces are not fixed.

MAIN LIMITATION OF BSM PUT OPM (cont'd)

- The price of the otherwise identical unrestricted asset will change during the restriction period due to normal market forces.
- The asset holder's loss of timing flexibility should be measured relative to the opportunity she would have to sell at any of these market prices were there no restrictions on the asset's transferability.

LOOKBACK PUT OPTION DLOM MODEL

- The lookback put OPM formula is:

$$D^*(T) = V_0\left[\left(2+\frac{\sigma^2 T}{2}\right)N\left(\frac{\sqrt{\sigma^2 T}}{2}\right)+\sqrt{\frac{\sigma^2 T}{2\pi}}exp\left(-\frac{\sigma^2 T}{8}\right)-1\right]$$

T=length of the restriction period
V_0=current share price
σ=volatility of the stock
N(·)=standard normal cdf

LOOKBACK PUT DLOM MODEL ASSUMPTIONS

- The investor is assumed to have perfect market-timing ability.
- In the absence of the marketability restrictions, the investor is assumed to be able to identify the maximum price during the restriction period – even though it has not ended yet – and to be able to sell the shares immediately.

LOOKBACK PUT DLOM MODEL ASSUMPTIONS (cont'd)

- Assumes the standard BSM share price equation.
- The length of the restriction period is fixed.
- Zero dividends.
- The lookback put OPM measures the loss of resale flexibility with respect to the maximum price of the unrestricted shares during the restriction period.

AVERAGE-STRIKE PUT OPTION DLOM MODEL

- The average-strike put OPM formula is:

$$D(T) = V_0 e^{-qT}\left[N\left(\frac{v\sqrt{T}}{2}\right) - N\left(-\frac{v\sqrt{T}}{2}\right)\right]$$

$$v\sqrt{T} = \left[\sigma^2 T + \ln\left(2\left\{e^{\sigma^2 T} - \sigma^2 T - 1\right\}\right) - 2\ln\left(e^{\sigma^2 T} - 1\right)\right]^{1/2}$$

T=length of the restriction period
V_0=current share price
σ=volatility of the stock
q=dividend yield (constant)
N(·)=standard normal cdf

AVERAGE-STRIKE PUT DLOM MODEL ASSUMPTIONS

- The investor is not assumed to have any special market-timing ability.
- Absent the marketability restrictions, investor is assumed to be equally likely to sell the shares any time during the restriction period.
- Distribution of the sum of independent lognormal random variables is approximately lognormal with the same first two moments.

AVERAGE-STRIKE PUT DLOM MODEL ASSUMPTIONS (cont'd)

- Assumes the standard BSM share price equation.
- The length of the restriction period is fixed.
- The average-strike put OPM measures the loss of resale flexibility with respect to the average price of the unrestricted shares during the restriction period.

AVERAGE-STRIKE PUT OPM MARKETABILITY DISCOUNTS

	q = 0											
Restriction Period	**σ = 0.1**	**σ = 0.2**	**σ = 0.3**	**σ = 0.4**	**σ = 0.5**	**σ = 0.6**	**σ = 0.7**	**σ = 0.8**	**σ = 0.9**	**σ = 1.0**	**σ = 1.1**	**σ = 1.2**
3 months	1.15%	2.30%	3.45%	4.59%	5.72%	6.85%	7.97%	9.07%	10.16%	11.24%	12.30%	13.34%
6 months	1.63	3.25	4.86	6.46	8.04	9.60	11.13	12.63	14.09	15.50	16.88	18.20
9 months	1.99	3.98	5.94	7.89	9.79	11.66	13.47	15.22	16.91	18.52	20.05	21.49
1 year	2.30	4.59	6.85	9.07	11.24	13.34	15.36	17.30	19.13	20.85	22.45	23.93
2 years	3.25	6.46	9.60	12.63	15.50	18.20	20.68	22.93	24.92	26.63	28.07	29.25
3 years	3.98	7.89	11.66	15.22	18.52	21.49	24.10	26.31	28.10	29.50	30.52	31.23
4 years	4.59	9.07	13.34	17.30	20.85	23.93	26.47	28.47	29.94	30.95	31.58	31.94
5 years	5.13	10.10	14.78	19.03	22.73	25.79	28.16	29.87	31.00	31.66	32.01	32.18

TESTING THE MARKETABILITY DISCOUNT MODEL

- I find evidence that transfer restriction effects, information and ownership concentration effects, and signally effects all contribute to the size of actual private placement discounts.
- The regression results enable us to remove the non-marketability-related effects to isolate on the transfer restriction effects.
- The resulting discounts are consistent with my model.

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IMPLIED TRANSFERABILITY DISCOUNT VS. MODEL PREDICTED DISCOUNT FOR DIFFERENT VOLATILITIES

Panel A: Offerings Announced Prior to February 1997

Volatility Range (%)	N	Mean Implied Marketability Discount (day prior)	Mean Model-Predicted Discount (T = 2)	Mean Model-Predicted Discount (T = 3)	Mean Model-Predicted Discount (T = 4)
0.0 - 29.9	4	19.47%	7.84%	9.54%	10.95%
30.0 - 44.9	0	-	-	-	-
45.0 - 59.9	4	10.51	16.97	20.15	22.55
60.0 - 74.9	9	13.82	19.18	22.03	23.96
75.0 - 89.9	10	24.15	23.16	26.52	28.64
90.0 - 104.9	2	34.97	26.10	29.07	30.64
105.0 - 120.0	1	61.51	28.40	30.73	31.70
> 120.0	6	44.10	30.88	31.95	32.20
Average:	**36**	**24.50%**	**21.37%**	**23.97%**	**25.62%**

Notes: The Model-Predicted Discount is calculated from the average-strike put option model (10) - (11), and the Implied Marketability Discount is calculated by subtracting the values of the control variables on the right-hand side of equation (16) from the observed Discount (day prior) for each sample private placement. Panel A applies to those offerings that were announced prior to February 1997. The sample includes 182 U.S. private placements between April 1, 1991 and March 8, 2007 that were priced at a discount. Twenty-six observations had to be dropped from the sample of 208 discounted stock private placements because 9 firms' historical proxy statements could not be obtained and the book value of equity of 17 other firms is negative.

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IMPLIED TRANSFERABILITY DISCOUNT VS. MODEL PREDICTED DISCOUNT FOR DIFFERENT VOLATILITIES

Panel B: Offerings Announced After February 1997

Volatility Range (%)	N	Mean Implied Marketability Discount (day prior)	Mean Model-Predicted Discount (T = 1)	Mean Model-Predicted Discount (T = 2)	Mean Model-Predicted Discount (T = 3)
0.0 - 29.9	7	12.66%	5.22%	6.99%	8.12%
30.0 - 44.9	19	18.56	7.95	10.75	12.64
45.0 - 59.9	17	15.92	11.74	16.15	19.22
60.0 - 74.9	28	19.21	14.83	19.84	23.00
75.0 - 89.9	25	21.37	17.82	23.50	26.83
90.0 - 104.9	16	21.61	20.28	26.07	29.05
105.0 - 120.0	6	24.89	22.74	28.29	30.65
> 120.0	28	29.71	27.72	31.20	32.02
Average:	**146**	**21.31%**	**17.02%**	**21.45%**	**23.86%**

Notes: The Model-Predicted Discount is calculated from the average-strike put option model (10) - (11), and the Implied Marketability Discount is calculated by subtracting the values of the control variables on the right-hand side of equation (16) from the observed Discount (day prior) for each sample private placement. Panel B applies to those offerings which were announced after February 1997. The sample includes 182 U.S. private placements between April 1, 1991 and March 8, 2007 that were priced at a discount. Twenty-six observations had to be dropped from the sample of 208 discounted stock private placements because 9 firms' historical proxy statements could not be obtained and the book value of equity of 17 other firms is negative.

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LIMITATION OF AVERAGE-STRIKE PUT OPM

- Average-strike put OPM tends to understate the DLOM when the stock price volatility is less than 45% or greater than 75%.
- It tends to overstate the DLOM when the stock price volatility is between 45% and 75%.

COMPARISON OF THE THREE PUT OPTION DLOM MODELS

- BSM put OPM is restrictive because of the fixed strike price assumption.
- Lookback put OPM is unrealistic because it assumes perfect market-timing ability.
- Average-strike put OPM provides an approximation because of the lognormality assumption but seems to fit empirically observed private placement discounts rather well.

Black-Scholes-Merton Model versus Average-Strike Put Option Model DLOM

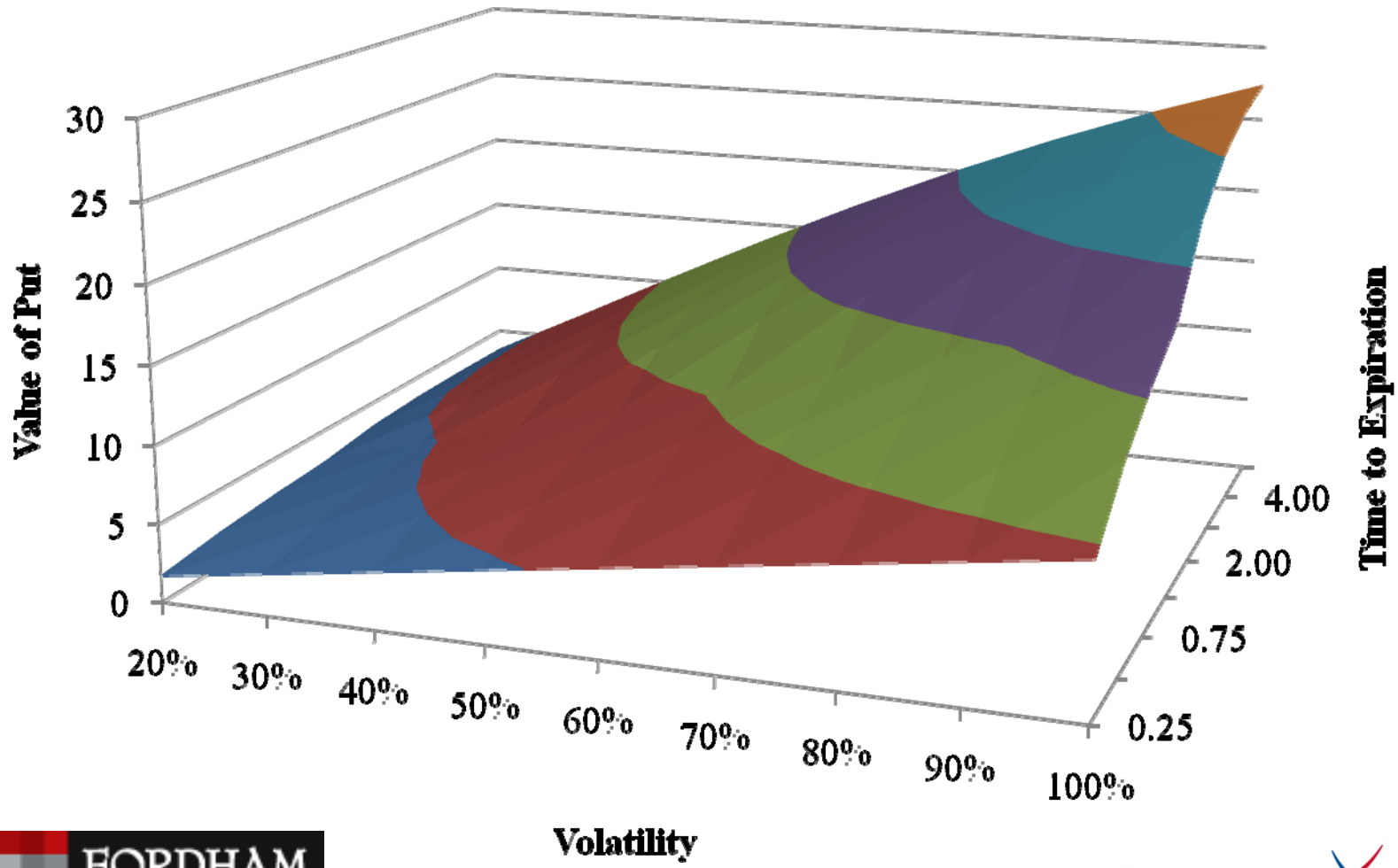
Panel A. Black-Scholes-Merton Put Option Value (q = 0)

(In Dollar Terms)

K	50
S_0	50
r	5%

T, σ	20%	30%	40%	50%	60%	70%	80%	90%	100%
0.25	1.6864	2.6704	3.6552	4.6385	5.6193	6.5967	7.5700	8.5386	9.5018
0.50	2.2099	3.5829	4.9580	6.3291	7.6930	9.0476	10.3909	11.7211	13.0367
0.75	2.5459	4.2051	5.8679	7.5235	9.1662	10.7920	12.3975	13.9797	15.5358
1.00	2.7868	4.6771	6.5729	8.4578	10.3231	12.1628	13.9720	15.7462	17.4816
2.00	3.3053	5.8387	8.3870	10.9057	13.3725	15.7722	18.0928	20.3247	22.4599
3.00	3.4976	6.4381	9.4044	12.3193	15.1446	17.8553	20.4332	22.8648	25.1400
4.00	3.5432	6.7611	10.0165	13.1970	16.2478	19.1348	21.8349	24.3325	26.6185
5.00	3.5093	6.9189	10.3782	13.7383	16.9281	19.9053	22.6436	25.1277	27.3516

Black-Scholes-Merton Model versus Average-Strike Put Option Model DLOM (cont'd)



Black-Scholes-Merton Model versus Average-Strike Put Option Model DLOM (cont'd)

Panel B1. Average-Strike Put Option Value (q = 0)
(In Percentage Terms)

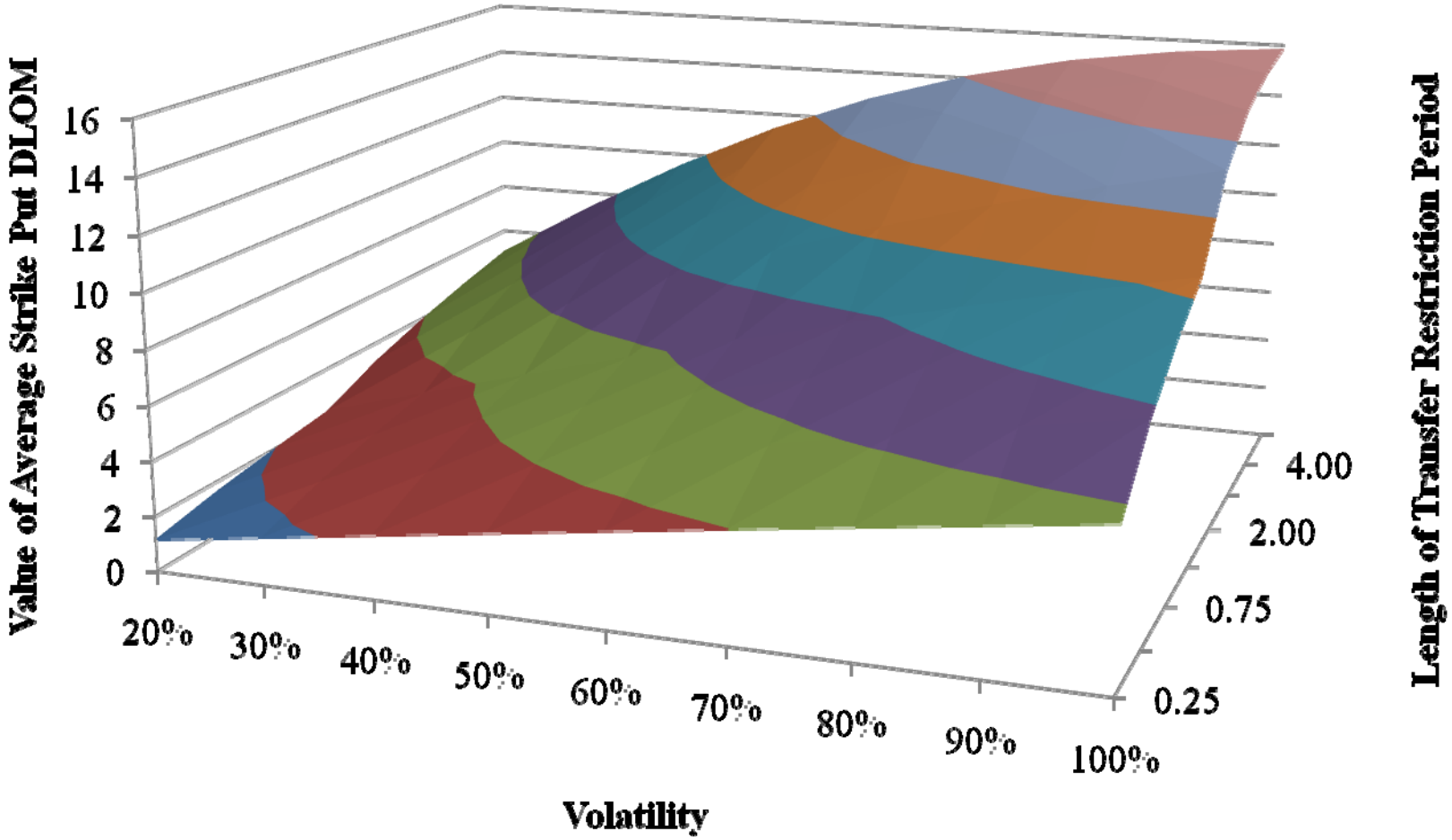
T, σ	20%	30%	40%	50%	60%	70%	80%	90%	100%
0.25	2.30%	3.45%	4.59%	5.72%	6.85%	7.97%	9.07%	10.16%	11.24%
0.50	3.25%	4.86%	6.46%	8.04%	9.60%	11.13%	12.63%	14.09%	15.50%
0.75	3.98%	5.94%	7.89%	9.79%	11.66%	13.47%	15.22%	16.91%	18.52%
1.00	4.59%	6.85%	9.07%	11.24%	13.34%	15.36%	17.30%	19.13%	20.85%
2.00	6.46%	9.60%	12.63%	15.50%	18.20%	20.68%	22.93%	24.92%	26.63%
3.00	7.89%	11.66%	15.22%	18.52%	21.49%	24.10%	26.31%	28.10%	29.50%
4.00	9.07%	13.34%	17.30%	20.85%	23.93%	26.47%	28.47%	29.94%	30.95%
5.00	10.10%	14.78%	19.03%	22.73%	25.79%	28.16%	29.87%	31.00%	31.66%

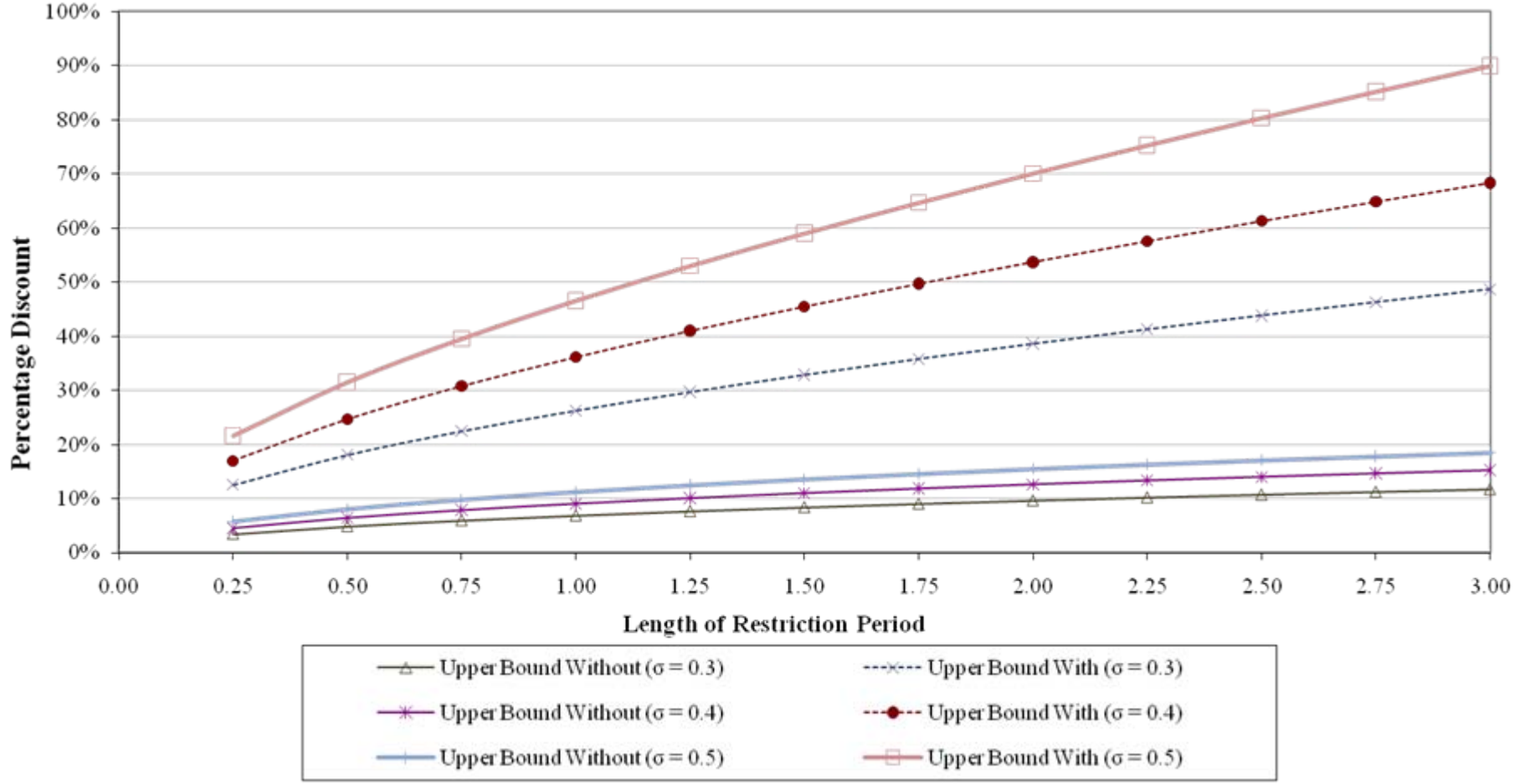
V_0	50

Panel B2. Average-Strike Put Option Value (q = 0)
(In Dollar Terms)

T, σ	20%	30%	40%	50%	60%	70%	80%	90%	100%
0.25	1.1505	1.7237	2.2943	2.8616	3.4248	3.9829	4.5352	5.0808	5.6191
0.50	1.6255	2.4323	3.2321	4.0223	4.8009	5.5653	6.3135	7.0431	7.7522
0.75	1.9889	2.9725	3.9430	4.8962	5.8280	6.7341	7.6107	8.4538	9.2598
1.00	2.2943	3.4248	4.5352	5.6191	6.6700	7.6819	8.6488	9.5651	10.4260
2.00	3.2321	4.8009	6.3135	7.7522	9.1001	10.3421	11.4650	12.4587	13.3167
3.00	3.9430	5.8280	7.6107	9.2598	10.7475	12.0509	13.1544	14.0521	14.7490
4.00	4.5352	6.6700	8.6488	10.4260	11.9638	13.2367	14.2354	14.9708	15.4738
5.00	5.0506	7.3910	9.5155	11.3645	12.8935	14.0813	14.9363	15.4984	15.8312

Black-Scholes-Merton Model versus Average-Strike Put Option Model DLOM (cont'd)





Comparison of the Marketability Discounts for Investors With and Without Special Timing Ability.
The figure compares the upper bound on the marketability discount implied by the unadjusted lookback put option model (Longstaff, 1995), which assumes perfect timing ability (Upper Bound With), and the upper bound on the discount obtained from the average-strike put option model, which assumes no special timing ability (Upper Bound Without), when $\sigma = 0.3$, $\sigma = 0.4$, $\sigma = 0.5$. Since Longstaff's model assumes a non-dividend-paying stock, q is set to zero in the average-strike put option model.

Lookback Put Option Model DLOM versus Average-Strike Put Option Model DLOM

Panel A1. Lookback Put Option Value (q = 0)
(In Percentage Terms)

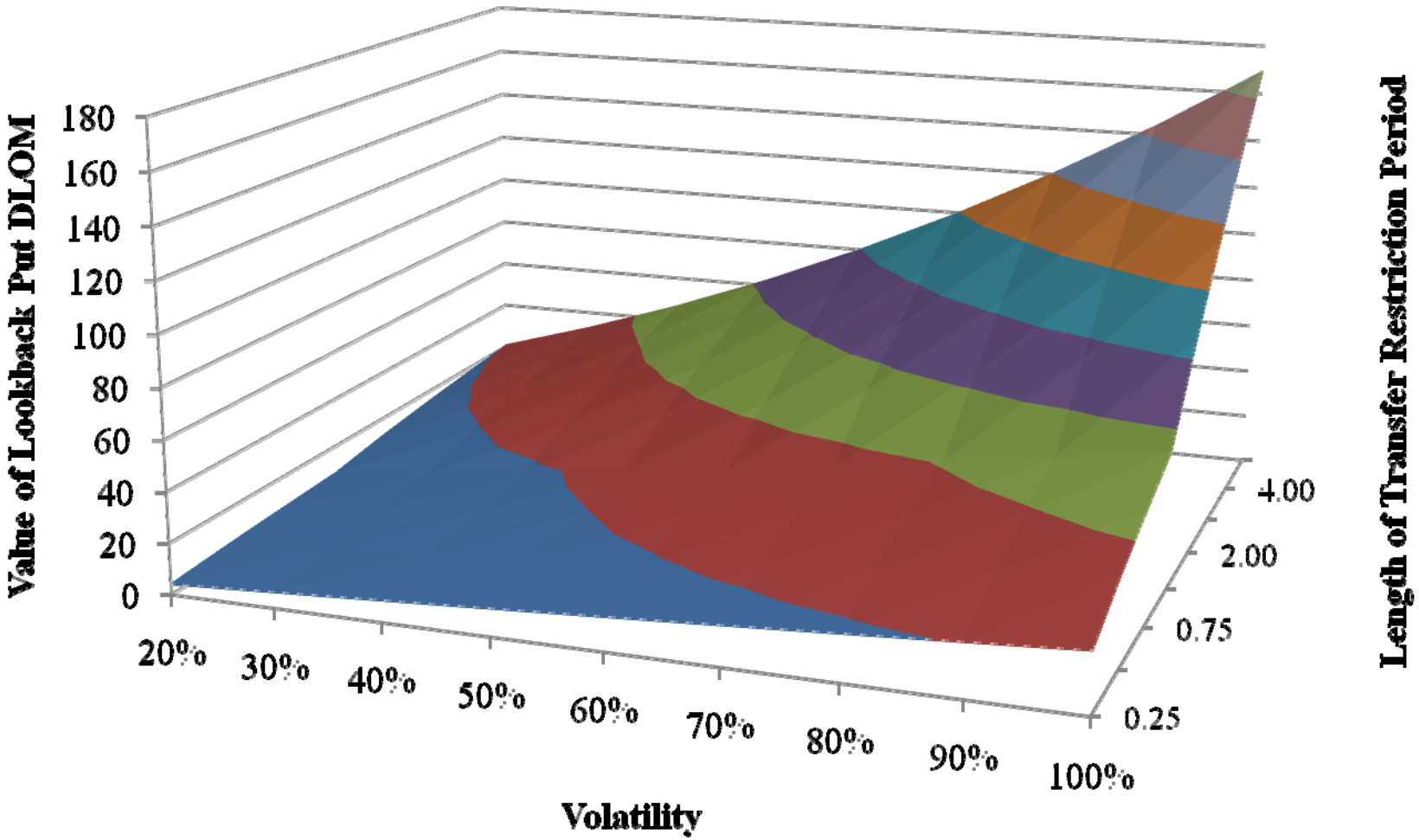
T, σ	20%	30%	40%	50%	60%	70%	80%	90%	100%
0.25	8.23%	12.54%	16.98%	21.56%	26.28%	31.13%	36.13%	41.27%	46.56%
0.50	11.79%	18.08%	24.64%	31.48%	38.60%	46.02%	53.73%	61.75%	70.09%
0.75	14.59%	22.48%	30.78%	39.51%	48.67%	58.29%	68.38%	78.94%	89.99%
1.00	16.98%	26.28%	36.13%	46.56%	57.59%	69.24%	81.52%	94.46%	>100%
2.00	24.64%	38.60%	53.73%	70.09%	87.72%	>100%	>100%	>100%	>100%
3.00	30.78%	48.67%	68.38%	89.99%	>100%	>100%	>100%	>100%	>100%
4.00	36.13%	57.59%	81.52%	>100%	>100%	>100%	>100%	>100%	>100%
5.00	40.98%	65.77%	93.72%	>100%	>100%	>100%	>100%	>100%	>100%

V_0	50

Panel A2. Lookback Put Option Value (q = 0)
(In Dollar Terms)

T, σ	20%	30%	40%	50%	60%	70%	80%	90%	100%
0.25	4.1161	6.2710	8.4921	10.7808	13.1381	15.5654	18.0639	20.6347	23.2793
0.50	5.8966	9.0412	12.3214	15.7406	19.3023	23.0101	26.8673	30.8773	35.0435
0.75	7.2935	11.2377	15.3888	19.7531	24.3371	29.1470	34.1890	39.4694	44.9941
1.00	8.4921	13.1381	18.0639	23.2793	28.7940	34.6177	40.7597	47.2295	**54.0361**
2.00	12.3214	19.3023	26.8673	35.0435	43.8579	**53.3365**	**63.5047**	**74.3871**	**86.0071**
3.00	15.3888	24.3371	34.1890	44.9941	**56.8003**	**69.6537**	**83.5988**	**98.6773**	**114.9288**
4.00	18.0639	28.7940	40.7597	**54.0361**	**68.6951**	**84.8052**	**102.4310**	**121.6328**	**142.4660**
5.00	20.4895	32.8858	46.8602	**62.5157**	**79.9505**	**99.2568**	**120.5203**	**143.8192**	**169.2247**

Lookback Put Option Model DLOM versus Average-Strike Put Option Model DLOM (cont'd)



Lookback Put Option Model DLOM versus Average-Strike Put Option Model DLOM (cont'd)

Panel B1. Average-Strike Put Option Value (q = 0)
(In Percentage Terms)

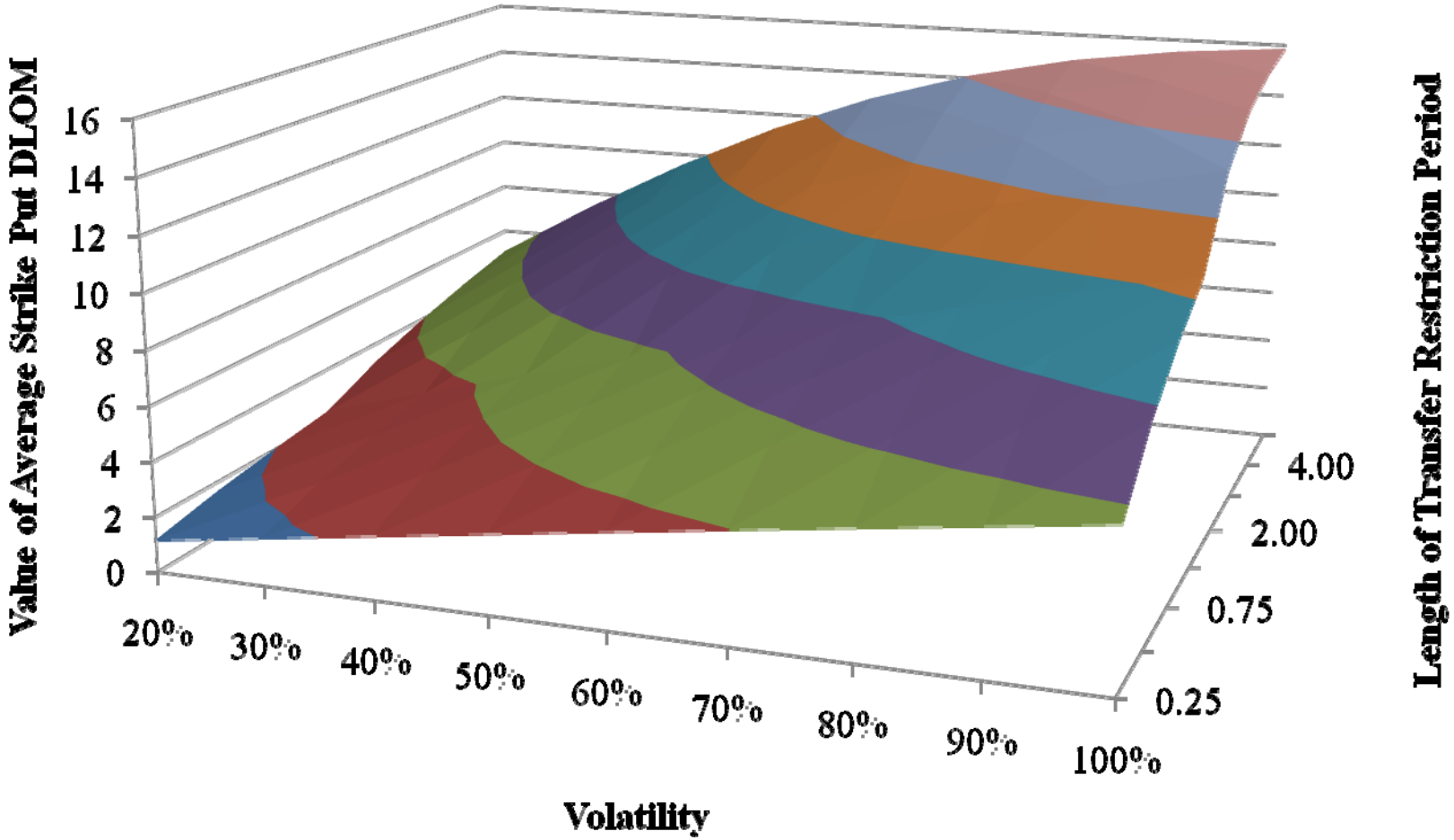
T, σ	20%	30%	40%	50%	60%	70%	80%	90%	100%
0.25	2.30%	3.45%	4.59%	5.72%	6.85%	7.97%	9.07%	10.16%	11.24%
0.50	3.25%	4.86%	6.46%	8.04%	9.60%	11.13%	12.63%	14.09%	15.50%
0.75	3.98%	5.94%	7.89%	9.79%	11.66%	13.47%	15.22%	16.91%	18.52%
1.00	4.59%	6.85%	9.07%	11.24%	13.34%	15.36%	17.30%	19.13%	20.85%
2.00	6.46%	9.60%	12.63%	15.50%	18.20%	20.68%	22.93%	24.92%	26.63%
3.00	7.89%	11.66%	15.22%	18.52%	21.49%	24.10%	26.31%	28.10%	29.50%
4.00	9.07%	13.34%	17.30%	20.85%	23.93%	26.47%	28.47%	29.94%	30.95%
5.00	10.10%	14.78%	19.03%	22.73%	25.79%	28.16%	29.87%	31.00%	31.66%

V_0	50

Panel B2. Average-Strike Put Option Value (q = 0)
(In Dollar Terms)

T, σ	20%	30%	40%	50%	60%	70%	80%	90%	100%
0.25	1.1505	1.7237	2.2943	2.8616	3.4248	3.9829	4.5352	5.0808	5.6191
0.50	1.6255	2.4323	3.2321	4.0223	4.8009	5.5653	6.3135	7.0431	7.7522
0.75	1.9889	2.9725	3.9430	4.8962	5.8280	6.7341	7.6107	8.4538	9.2598
1.00	2.2943	3.4248	4.5352	5.6191	6.6700	7.6819	8.6488	9.5651	10.4260
2.00	3.2321	4.8009	6.3135	7.7522	9.1001	10.3421	11.4650	12.4587	13.3167
3.00	3.9430	5.8280	7.6107	9.2598	10.7475	12.0509	13.1544	14.0521	14.7490
4.00	4.5352	6.6700	8.6488	10.4260	11.9638	13.2367	14.2354	14.9708	15.4738
5.00	5.0506	7.3910	9.5155	11.3645	12.8935	14.0813	14.9363	15.4984	15.8312

Lookback Put Option Model DLOM versus Average-Strike Put Option Model DLOM (cont'd)



EXAMPLE
Fuel Systems Solutions, Inc.
Private Placement Discount

Terms and Parameters	
Offer Date	12/18/2003
Offer Price	$6.40
Closing Price (day of offering)	8.13
Closing Price (day prior)	7.37
Shares Sold	1,500,000
Shares Outstanding Prior to Offering	17,003,000
Average Weekly Trading Volume (4 weeks' historical data)	1,141,194
1% of Shares Outstanding	170,030
Offering Gross Proceeds	9,600,000
Rule 144 Minimum Holding Period (years)	1.000
Average Block Size (years to sell)	1.103[1]
Total Restriction Period (years)	2.103
Volatility (24 months' historical data)	88%
Dividend Yield	0%

[1]Average Block Size (years to sell) = 1/2 * Shares Sold in Offering / [Min (Avg Weekly Trading Volume Four Weeks Prior to Offering, 1% of Shares Outstanding) * 4].

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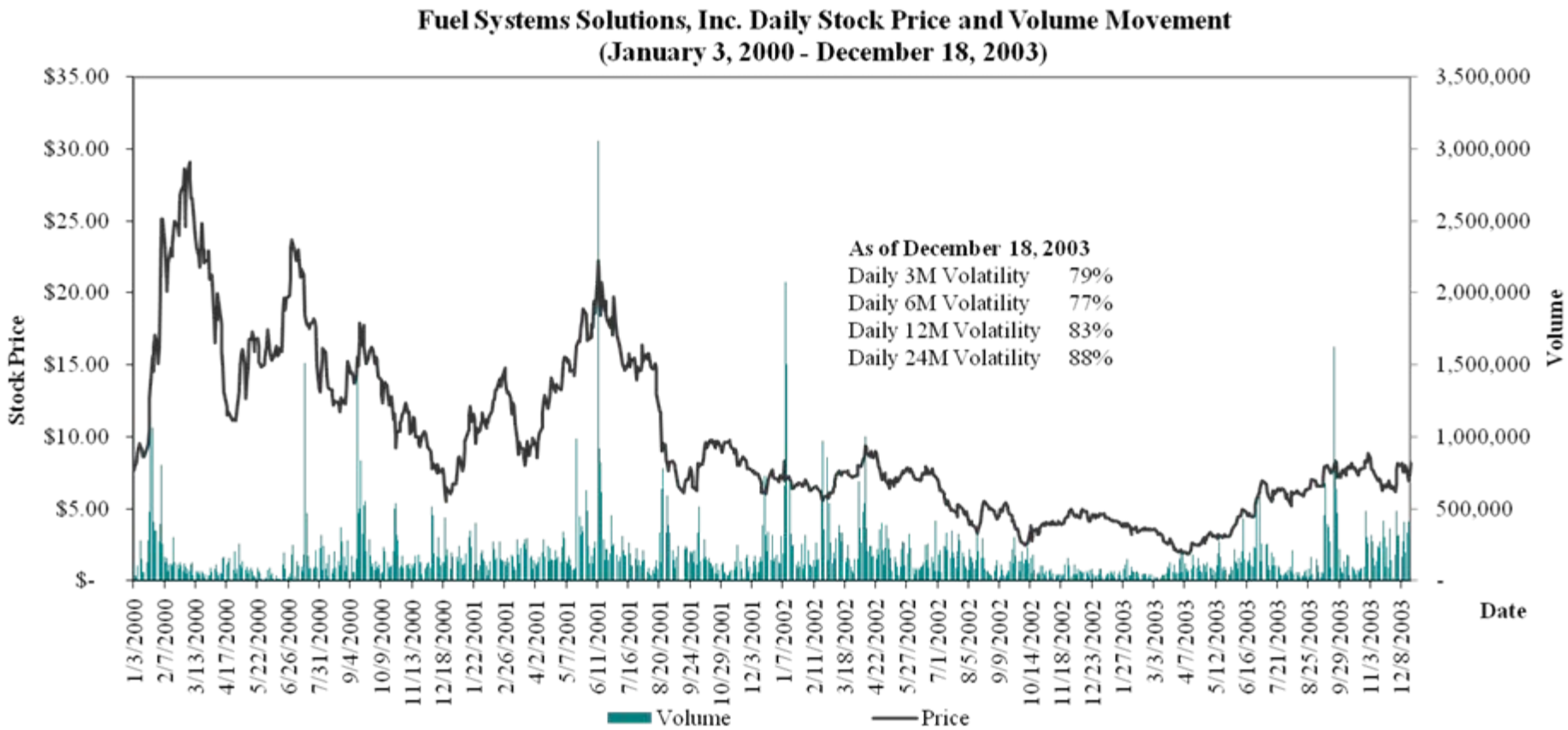
EXAMPLE
Fuel Systems Solutions, Inc.
Private Placement Discount



Source: Bloomberg, L.P.

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EXAMPLE
Fuel Systems Solutions, Inc.
Private Placement Discount

Effect of Control Variables

	Coefficient	Value	Estimate
Intercept	9.190		8.9%
Model-Predicted Discount	0.759	25.000	19.0%
Risk	0.552	0.429	0.2%
Fraction	1.880	0.081	0.2%
Exchange	-4.666	1.000	-4.7%
Direct	1.901	2.397	4.6%
Market / Book Deviation	2.397	1.500	3.6%
Post	-0.170	1.000	-0.2%
Options	-0.443	0.000	0.0%
Registration Rights	-1.821	1.000	-1.8%
Registered Stock	-6.502	0.000	0.0%
Implied Transferability Discount			**29.8%**

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EXAMPLE
Fuel Systems Solutions, Inc.
Private Placement Discount

Marketability Discount	
Discount (day of offering)	21.3%
Discount (day prior)	13.2%
Implied Transferability Discount	29.8%
Average Strike Put Model-Predicted Discount (Slide 6)	25.0%
Unadjusted Lookback Put Model-Predicted Discount	> 100.0%
Black-Scholes-Merton Put Model Predicted Discount (r = 1.24% on day of offering)	45.8%

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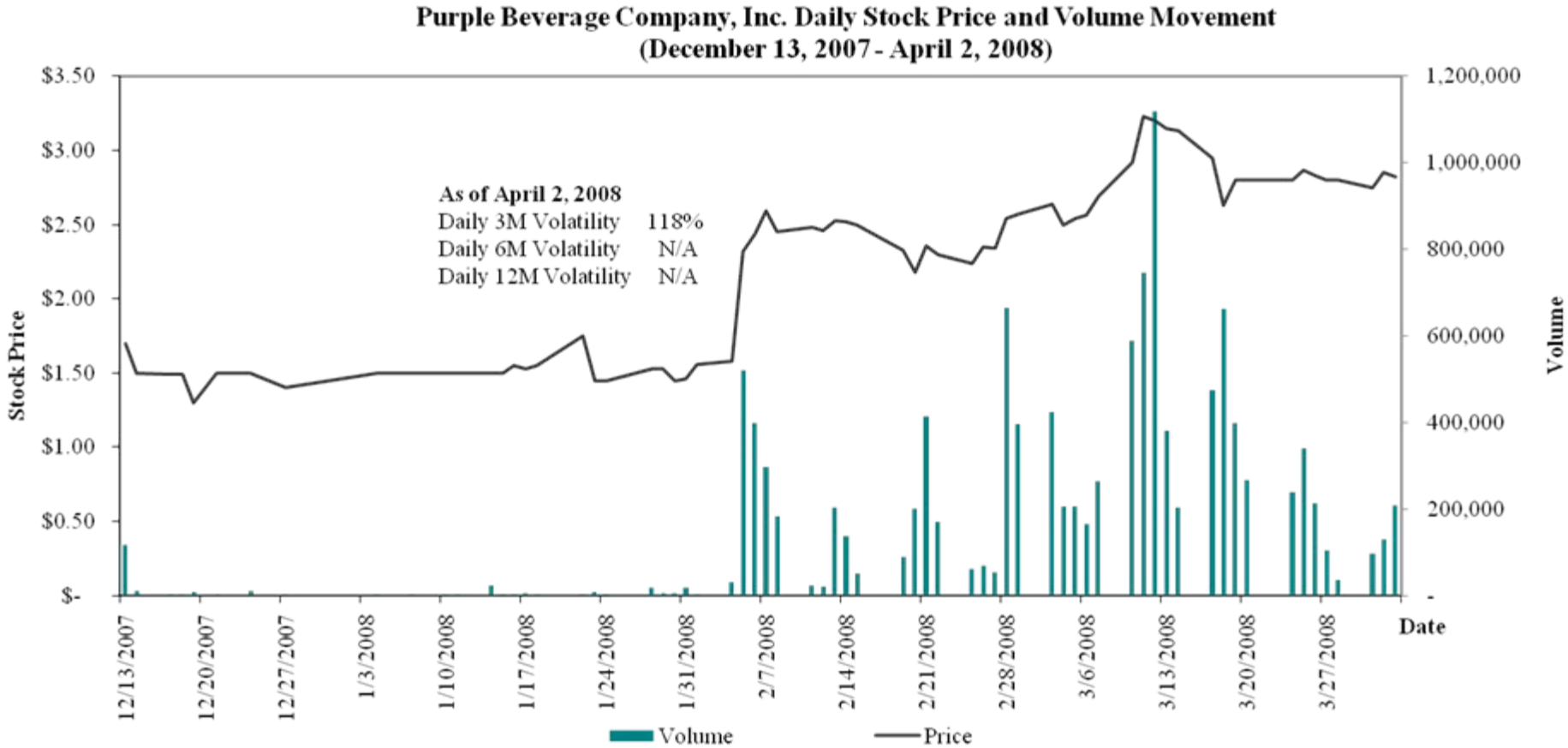
EXAMPLE
Purple Beverage Company, Inc.
Private Placement Discount

Terms and Parameters	
Offer Date	4/2/2008
Offer Price	$1.40
Closing Price (day of offering)	2.82
Closing Price (day prior)	2.85
Shares Sold	1,635,714
Shares Outstanding Prior to Offering	55,261,000
Average Weekly Trading Volume (3 months' historical data)	705,889
20% of Average Weekly Trading Volume	141,178
Offering Gross Proceeds	2,290,000
Rule 144 Minimum Holding Period (years)	0.500
Average Block Size (years to sell)	0.111[1]
Total Restriction Period (years)	0.611
Volatility (3 months' historical data)	118%
Dividend Yield	0%

[1]Average Block Size (years to sell) = 1/2 * Shares Sold in Offering / (0.2 x Average Weekly Trading Volume * 52].

EXAMPLE
Purple Beverage Company, Inc.
Private Placement Discount



Source: Bloomberg, L.P.

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EXAMPLE
Purple Beverage Company, Inc.
Private Placement Discount

Effect of Control Variables

	Coefficient	Value	Estimate
Intercept	9.190		8.90%
Model-Predicted Discount	0.759	19.500	14.80%
Risk	0.552	0.571	0.32%
Fraction	1.880	0.029	0.05%
Exchange	-4.666	0.000	0.00%
Direct	1.901	N/A	N/A
Market / Book Deviation	2.397	1.250	3.00%
Post	-0.170	1.000	-0.17%
Options	-0.443	0.000	0.00%
Registration Rights	-1.821	1.000	-1.82%
Registered Stock	-6.502	0.000	0.00%
Implied Transferability Discount			**25.1%**

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EXAMPLE
Purple Beverage Company, Inc.
Private Placement Discount

Marketability Discount	
Discount (day of offering)	50.4%
Discount (day prior)	50.9%
Implied Transferability Discount	25.1%
Average Strike Put Model-Predicted Discount (Slide 6)	19.5%
Unadjusted Lookback Put Model-Predicted Discount	97.4%
Black-Scholes-Merton Put Model Predicted Discount (r = 1.67% on day of offering)	34.9%

CONCLUSIONS

- Marketability restrictions can reasonably be priced as the value of an average-strike put option.
- The marketability discount varies directly with the volatility of the stock, as the average-strike OPM predicts.
- A marketability discount is different from a liquidity discount.
- A marketability discount model can be used to estimate a liquidity discount provided the parameter values are chosen carefully.