Collars, Prepaid Forwards, and the DLOM: Volatility Is the Missing Link

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The variable prepaid forward (VPF) model assumes that a marketability restriction only costs the asset owner the time value of money during the restriction period. It does not fit the definition of the marketability discount. A put-option model is better suited for the discount for lack of marketability (DLOM) calculation. The VPF model will usually significantly underestimate the DLOM, the more so the higher the asset’s price volatility.

Introduction

The proper method for calculating the discount for lack of marketability (DLOM) continues to be debated in the appraisal literature. Several papers have recognized the optionality inherent in the loss of selling flexibility when a stock has marketability restrictions and have proposed different put-option models to calculate the DLOM. These models have been criticized for implying DLOMs that are supposedly “too large” as compared to the discounts observed in practice. Abbott (2009) recommends an adjustment to the put-option formulas to reduce the purported overstatement, and Fishman and Barenbaum (2013) champion an entirely different approach that models the DLOM as the price of a variable prepaid forward (VPF) contract.

A paper by Finnerty (2013b) disagrees with Abbott’s (2009) proposed “fix” in general but identifies circumstances when it might be appropriate. This article takes issue with applying the VPF model. In a nutshell, Fishman and Barenbaum’s (2013) recommended VPF approach has a fatal flaw. It calculates a DLOM that is very sensitive to the interest rate in the VPF contract but is insensitive to, and in some cases entirely independent of, the stock’s volatility. This is inconsistent with economic intuition and empirical evidence (Finnerty 2012, 2013a), which suggest just the opposite: The DLOM should be sensitive to a stock’s volatility but insensitive to the interest rate.

Fishman and Barenbaum (2013) recommend replacing the put-option model with the VPF model as the basis for calculating DLOMs. However, their definition of the marketability discount is different from the conventional definition. They define the marketability discount as “the cost of preserving the asset value (the value were it not for the marketability restrictions) and the cost of monetizing the asset at the current market value,” and they explain that “the lack of marketability and liquidity refers to the inability of the seller to get the current price in a timely fashion at a low cost” (emphasis added). The DLOM measures the loss of value due to the loss of selling flexibility during the period the restrictions are in place. This is different from preserving the market value of the unrestricted asset at the beginning of the restriction period.

The DLOM and What It Measures

The marketability of a financial instrument has been defined as “the ability to quickly convert property to cash at minimal cost.” The DLOM has been defined as “an amount or percentage deducted from the value of an ownership interest to reflect the relative absence of marketability.” A marketability restriction reduces the value of an asset as compared to an otherwise identical...
fully marketable asset because it deprives the asset owner of timing flexibility during the restriction period. In particular, it prevents the asset owner from selling the asset to avoid an impending decline in price and redeploying the money in a better investment, as he would be free to do absent the restriction. This loss of timing flexibility exposes the asset owner to greater risk of loss. Depending on the asset’s price volatility, this risk of loss and the consequent cost may be quite significant. A DLOM is typically applied when computing the fair market value of restricted stock, interests in privately held firms, and other assets where there is an absolute prohibition on sale (as opposed to mere difficulty in selling an illiquid asset).\(^8\)

The quoted definition of marketability actually lumps it together with liquidity. However, it is important to distinguish lack of liquidity from lack of marketability.\(^9\) Marketability refers to the asset holder’s legal and contractual ability to sell or otherwise transfer asset ownership. Lack of marketability occurs when legal or contractual restrictions on transfer prevent, or at least severely impair, the holder’s ability to sell the asset or transfer it until the restriction period lapses. For example, the almost absolute prohibition on transfer that typifies employee stock options or company awards of restricted stock illustrates a contractual restriction on marketability. The loss of resale or transfer flexibility gives rise to a loss of timing flexibility because the sale has to be postponed until the restrictions lapse. This loss of flexibility entails a cost, the DLOM, which can be modeled as the value of a put option.\(^10\)

Lack of registration under the securities laws is another example of a transfer restriction. However, even when securities are unregistered, in most cases there is not, strictly speaking, an absolute prohibition on transfer. A security holder can still transfer unregistered securities by qualifying for one of the exemptions from registration under the Securities Act of 1933. However, the buyer of unregistered shares also faces the same restrictions as the seller. So the assumption that the original holder cannot transfer the security may still be a reasonable assumption, even though it is not always absolutely correct.\(^11\)

Liquidity is different from marketability, although it, too, refers to an asset holder’s ease of transferring asset ownership. Liquidity is the relative ease with which an asset holder can convert the asset into cash without sacrificing any portion of the asset’s intrinsic value. A liquid market gives an asset holder the flexibility to sell an asset at any time of her choosing without any loss of intrinsic value. Lack of liquidity entails a loss of timing flexibility because the asset holder cannot dispose of the asset quickly unless he or she is willing to accept a significant reduction in value. An asset that lacks marketability also typically lacks liquidity because the legal or contractual restrictions on marketability inhibit the holder from selling or transferring the asset quickly for full value.

One can also model the cost of the lack of liquidity as the value of a forgone put option. However, the option formulation is more complex than in the case of lack of marketability because there is no legal or contractual restriction on the holder’s ability to sell or transfer the asset, and, consequently, the length of the restriction period is less clear.\(^12\) For example, the market for an asset may be poorly developed, making it difficult, time-consuming, and therefore expensive to find a buyer for the securities, but the assets are nevertheless marketable. The restrictions are financial, rather than legal or contractual, and there is no fixed date on which they are scheduled to lapse. It takes more time to find a buyer in an illiquid market than in a liquid market. This loss of flexibility to sell an asset freely, or equivalently, the ability to sell it quickly but only if there is some concession of intrinsic value, imposes a cost that can be modeled as the value of a put option. Using a put-option model to calculate a discount for lack of liquidity therefore requires more judgment than a DLOM.\(^13\)

**Put-Option Models for Calculating the DLOM**

One way for the owner of a restricted asset to eliminate its exposure to the risk of a future decrease in asset price is to purchase an option to sell the asset at a defined strike price that lasts until the end of the restriction period (a “put option”). There are several option pricing models that measure the price risk associated with the lack of marketability, all of which are based on the cost of buying put options to hedge the risk of a price decline during the period the asset is restricted. For instance, Longstaff (1995) derives an upper bound on the DLOM by modeling the value of marketability as the price of a look-back put option. This model assumes that investors have perfect market-timing ability. Finnerty (2012)\(^14\)

\(^9\)See Finnerty (2013b) for a fuller explanation of the difference.
\(^10\)See the sources cited in footnote 2.
\(^11\)To the extent there are viable secondary markets for unregistered stock, they transform a security that essentially lacks marketability into one that lacks liquidity. Thus, lack of marketability and lack of liquidity involve similar losses of resale or transfer flexibility but with different root causes.
\(^12\)Even with marketability discounts, the length of the restriction period must be adjusted upward to reflect how long it is expected to take to sell all the shares after their resale restrictions lapse. The larger the block of shares, the longer it is likely to take to sell them, and the greater should be the length of the assumed restriction period.
\(^13\)While Finnerty (2012) and Longstaff (1995) address discounts for lack of marketability, Abbott (2009) is concerned with discounts for lack of liquidity.

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models the DLOM as the value of an average-strike put option and confirms empirically that the model-predicted DLOMs are consistent with actual private placement discounts after adjusting for the equity ownership concentration, information gathering, and expected underperformance postissue effects that typically accompany a private placement. These put-option models were developed to quantify the DLOM to compensate the asset holder for the loss of timing flexibility inherent in any marketability restrictions that apply over the entire length of the restriction period.

**Overview of Variable Prepaid Forward Contracts**

A recent article by Fishman and Barenbaum (2013) criticizes the use of put-option models to estimate the DLOM and, instead, recommends performing the calculation based on an at-the-money collar strategy utilizing a VPF contract. A VPF is an over-the-counter forward contract. The seller of a VPF agrees to deliver a predetermined number of shares to the purchaser, typically a brokerage firm, at a specified future date and receives in return a stated percentage of the value of the shares when the contract is executed. The term variable refers to the fact that the number of shares to be delivered at maturity depends on the value of the stock at that time. The term prepaid refers to the up-front payment.

A VPF has four components: shares of common stock, the purchase of a put option, the sale of a call option, and an up-front cash prepayment for the shares. The VPF seller effectively incurs a loan against the future delivery of the underlying shares. The amount of the borrowing corresponds to the fair market value of the shares minus the net cost of the two options (including any transaction costs) and minus the interest and other costs of the loan. The combination of a long put option and a short call option creates what is known as a collar. The strike price of the put option puts a floor below the current sale price for the collared shares, and the strike price of the call option places a cap on this future sale price. The floor and the cap constrain the number of shares, or the equivalent amount of cash, to be delivered at maturity. By combining the call and put options, an investor is able to protect her investment against downside risk below the floor while enjoying upside potential up to the cap. Setting the strike prices of the put and call options equal to the current price of the stock, as Fishman and Barenbaum (2013) assume, creates an at-the-money collar, which guarantees that the investor will receive the amount of the strike price per share for all the shares delivered at contract expiration.

The up-front payment is economically equivalent to the amount of money the VPF seller borrows against the future delivery of the shares. The up-front payment the investor receives is less than the value of the shares to be delivered, and the discount (referred to as the ‘‘borrowing cost’’ when expressed on a per-annum basis) reflects the transaction costs and the dealer’s profit. When the strike prices of the two options match the current share price, the discount implicit in the up-front payment has four basic components: the interest on the prepayment amount during the term of the VPF contract plus the net cost to buy the put option and sell the call option plus any other hedging costs the dealer might incur plus the dealer’s profit. Because the VPF seller receives a cash payment now and is able to defer capital gains taxes until the transfer is complete, VPFs are often used by investors to lock in their profits now and to defer their capital gains taxes on low-tax-basis common stocks. The main purpose of the VPF is to defer capital gains taxes; the liquidity benefit is secondary.

For example, consider an investor who purchased 1,000 shares of XYZ common stock for $30 per share, totaling $30,000. Assume that the investor would like to sell the stock, but the current stock price is $100. The investor would have to pay capital gains tax on $70,000 (= 1,000 \times \$100 - \$300). Alternatively, the investor could enter into a VPF with a brokerage firm under which he would deliver 1,000 shares in three years and, assuming a 20% discount, receive an up-front payment of $80,000 (= 100,000 \times 80\%). Suppose that the floor is $90 (90% of the stock price), and the cap is $120 (120% of the stock price). The investor remains exposed to the price risk of the stock for price changes between the floor and the cap. On the collar expiration date, depending on the stock price on that date, the investor will either deliver a variable number of shares to settle the transaction (physical settlement) or pay in cash (cash settlement).

**Modeling the DLOM as the Price of a VPF Contract**

Fishman and Barenbaum’s (2013) definition of the DLOM is different from the more conventional definition stated earlier in this article in that their DLOM assumes

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14David Tabak (2003) also recommends the VPF approach, which he refers to as a cashless collar strategy.

15The discount is typically negotiated between the investor and the dealer. We are not aware of any publicly available database that provides information concerning the components of the discount.

that the current market value of the unrestricted asset at the beginning of the restriction period is preserved.\textsuperscript{17} However, the DLOM should instead measure the cost attributable to the loss of selling flexibility during the entire restriction period.

Fishman and Barenbaum (2013) argue that put-option models of the DLOM overstate the cost of preserving the current market value because put options protect an equity holder’s downside risk but do not limit the investor’s upside potential. Specifically, they note that the cost of put options reflects the cost of both benefits—limiting the downside risk and maintaining the potential for any price appreciation in the future. Thus, an investor who owns stock and has purchased a put option struck at the current price is in a better position than when he sells the stock at the current price. Therefore, the article asserts that an at-the-money collar, which subtracts the price of the call option, is a more appropriate model for calculating a DLOM.

However, the at-the-money collar strategy assumes the forward sale of the shares, rather than the loss of selling flexibility over a time interval. In a nutshell, the VPF methodology assumes that the asset owner can avoid the effect of the restrictions on the sale of the asset by entering into a collar strategy, which guarantees him receipt of the full value of that ownership interest at maturity, with the only costs to the asset owner being the interest on the VPF proceeds. Specifically, they state, ‘‘therefore, the interest cost of the prepaid variable forward is the best proxy for the marketability discounts.’’\textsuperscript{18} As a result, the cost of the VPF is the interest cost of the contract, which can significantly understated the DLOM, as illustrated by the following example.

**Example**

The VPF strategy works as illustrated in Table 1. An investor owns a share of Coca-Cola stock worth $40 but is restricted from selling that share for some period of time. He can enter into an at-the-money collar strategy to guarantee his receipt of exactly $40 at the end of the restriction period. That option strategy, which is tantamount to selling a forward contract on Coca-Cola stock, involves buying a put option and selling a call option on each share of Coca-Cola stock. The pair of options forms a ‘‘collar,’’ because the classic use of the strategy is to limit, or ‘‘collar,’’ the investor’s ultimate gain or loss within a tight band.

In the Fishman and Barenbaum (2013) VPF, both options have the same ‘‘strike price’’ of $40, which means the collar is as tight as it can be. It guarantees the investor’s receipt of exactly $40 at the end of the period. Thus, there is no price risk. Note that the investor can execute the collar strategy at a profit when the stock is non-dividend-paying because the investor can sell the call option on a share of Coca-Cola stock for a higher price than he will have to pay for the put option on a share of Coca-Cola stock.\textsuperscript{19} The examples in Panel A of Table 1 illustrate that when put-call parity holds, and the Black-Scholes-Merton model is used to calculate the option prices, the call price exceeds the put price, which reduces the cost of the collar and thus the DLOM.\textsuperscript{20}

Next, the owner of Coca-Cola stock borrows against the guaranteed future receipt of $40. Suppose the borrowing cost is 5% per year, and the loan is for one year.\textsuperscript{21} The investor is selling the stock one year ‘‘forward.’’ Since he is borrowing against the $40 future sales proceeds, the $40 sales price is effectively ‘‘prepaid’’ to him less interest on the loan. The investor’s cost of this strategy—paying interest at 5% for one year—will be partially offset due to any profit on the collar, also as illustrated in Table 1. The investor agrees to pay $1.90 (=$40−[40/1.05]) of interest and receives net proceeds of $38.23 (=$40+0.13−1.90). The VPF model DLOM is 4.43%.

Note that the put-option DLOM is less than the VPF DLOM for Coca-Cola stock in Table 1. The average-strike put-option model DLOM is only 3.19%. Fishman and Barenbaum (2013) criticize the put-option model for overstating the DLOM, but their model calculates an even higher DLOM for the low-volatility stock. However, their model understates the DLOM for high-volatility stocks. Even worse, it calculates essentially the same DLOM for low-volatility and high-volatility stocks in Table 1 because it fails to take into account the effect that volatility has on the cost of the stock’s lack of marketability.

\textsuperscript{17}Fishman and Barenbaum (2013, page 10).
\textsuperscript{18}Fishman and Barenbaum (2013, page 11). This simplification ignores the net cost of the call and put options, any other hedging costs, and the profit for the broker-dealer for implementing the collar strategy. Tabak (2003) recommends a zero-cost collar in which the prices of the call and put options are equal, and the net cost is therefore zero, as a way of reducing a share owner’s marketability risk exposure.

\textsuperscript{19}This result is a direct consequence of put-call parity. See Hull (2012, pages 221–225).
\textsuperscript{20}For a dividend-paying stock, the present value of the dividends on the underlying stock expected to be paid during the remaining term of the options reduces the current stock price. Consequently, the price of the put option could exceed the price of the call option for a dividend-paying stock.
\textsuperscript{21}The 5% borrowing cost in the example is a hypothetical rate. Based on empirical research, the discount typically ranges from 10% to 20% of the current stock price depending on the term of the contract, which typically ranges from two years to five years. These features imply a borrowing cost of about 5% per annum. See Berman, Pike, and Corrao (2011) and Jagolinzer, Matsunaga, and Yeung (2007). The assumed borrowing cost thus falls within the customary range.
Flaws in the VPF Calculation of the DLOM

The preceding example illustrates three fundamental problems with using the VPF model to calculate DLOMs. First, the VPF DLOM model ignores the very important effect of stock price volatility on the DLOM. Even though some stocks are far more volatile than others, the VPF model is not able to adjust for the difference in stock volatility. The example in Table 1 illustrates this point. Coca-Cola has a very low volatility of 13.86%, while Twitter has a relatively high volatility of 50.21%. Because volatility dramatically increases the price of options, the put-option model DLOM can account for volatility. In an at-the-money collar, on the other hand, the put-option price and the call-option price largely offset each other, and thus the volatility effect is more or less neutralized.

Second, the VPF DLOM reflects the borrowing cost of the collar contract; it does not reflect the lack of marketability of the underlying assets. Marketability becomes irrelevant because the VPF methodology assumes the shares are immediately sold, although delivery of the shares is delayed until the VPF contract matures. As illustrated in Panel B of Table 1, when applying the same 5% borrowing cost to the shares of both Coca-Cola and Twitter with the same restriction period, the VPF DLOMs are essentially the same, even though one stock is more than three times more volatile than the other.

### Table 1
Comparison of the DLOM for High-Volatility and Low-Volatility Common Stocks Based on the VPF Model

| Panel A. Pricing of a Call, a Put, and a Collar Based on the Black-Scholes-Merton Model |
|-----------------------------------------------|-----------------------------------------------|
| The Coca-Cola Company | Twitter, Inc. |
| **Call Option** | **Put Option** | **Call Option** | **Put Option** |
| Valuation Date | June 12, 2014 | June 12, 2014 | June 12, 2014 | June 12, 2014 |
| Expiration Date | June 12, 2015 | June 12, 2015 | June 12, 2015 | June 12, 2015 |
| Current Stock Price ($S_0$) | $40.00 | $40.00 | $35.00 | $35.00 |
| Strike Price (K) | 40.00 | 40.00 | 35.00 | 35.00 |
| Risk-Free Rate ($R_f$) | 0.33% | 0.33% | 0.33% | 0.33% |
| Dividend Rate ($D_0$) | 0.00% | 0.00% | 0.00% | 0.00% |
| Volatility ($\sigma$) | 13.86% | 13.86% | 50.21% | 50.21% |
| Time to Maturity (T) | 1.00 | 1.00 | 1.00 | 1.00 |
| BSM Option Price | $2.27 | $2.14 | $6.98 | $6.87 |

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<th>Panel B. Calculation of VPF DLOM</th>
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Notes:

* The dividend rate is assumed to be zero to make the two examples comparable.

† The value of a collar is the difference between the value of the call option and the value of the put option.

‡ The borrowing cost is assumed to be 5% per annum. Based on empirical research, the discount typically ranges from 10% to 20% of the current stock price depending on the term of the contract, which typically ranges from two years to five years. These features imply a borrowing cost of about 5% per annum. See Berman, Pike, and Corrao (2011) and Jagolinzer, Matsunaga, and Yeung (2007).

§ The net cash flow is equal to the borrowed amount plus the value of the collar less the borrowing cost.

# Source: Finnerty (2012).
Third, there are circumstances where a DLOM is appropriate, but implementing the VPF collar strategy would be very expensive, if not impossible. If a stock seldom trades, option market makers will not write options on it, and it would be impossible to engage in an option collar strategy unless a close publicly traded substitute could be identified. Without a close substitute, the collar will not hedge the stock price risk effectively (there is basis risk), and the VPF model cannot apply. Even for publicly traded stocks with traded options written on them, the collar could be extremely expensive to put on and maintain when the underlying stocks are highly volatile or trade in an illiquid market, or if the market for the options is illiquid.

Using the VPF model to calculate the DLOM for an equity ownership interest in a portfolio of common stocks is similarly very difficult, if not impossible. The portfolio’s composition will change as the portfolio manager rebalances it. Rebalancing would make the VPF strategy impossible to implement unless the collar could be restructured instantaneously each time the portfolio composition changes. Initially, the asset owner would have to buy and sell different baskets of call and put options corresponding to each of the many stocks in the portfolio. Then, each time the portfolio rebalances, the asset owner would have to enter into offsetting options trades to cancel those collars that were no longer necessary due to liquidated positions and enter into new collars for the new stock positions. With each of these transactions, the buyer of the collar would have to pay the spread between the bid price and the ask price of the call and put options, in order to unwind the collars on each share of stock that the portfolio manager sells and to create new collars on each share of stock that the portfolio manager buys. If it were even possible as a technical matter to pursue such a complex strategy, it would entail substantial transaction costs during the restriction period. As a practical matter, a VPF strategy could not be conducted with a portfolio of common stocks for which composition changes over time unless a matching publicly traded index could be identified. If it cannot, then attempting to implement such a strategy would likely be prohibitively expensive, if it could be done at all.

Finally, we are not aware of any court that has accepted the VPF model for calculating the DLOM. In the only two instances we could find where it was used for calculating the DLOM in litigation, the court rejected it.22 In Litman v. United States, the Internal Revenue Service challenged the marketability discount the taxpayer had claimed for a very large block of restricted stock. The court rejected the taxpayer’s methodology because the transaction was “impossible as a practical matter,” and thus it “relies on real costs that have been shown not to be achievable.”23 The court held that because the block of restricted stock was so large compared to the public float, it was impossible in practice for the owner to buy and sell options of sufficient volume to undertake the VPF strategy. As a result, it was impossible for the asset holder to reduce the cost of the marketability restriction to the mere payment of interest.

**Conclusion**

In essence, the VPF model assumes that a marketability restriction only costs the asset owner the time value of money at a fixed interest rate (ignoring the other costs of implementing the VPF strategy) for the duration of the restriction period. The VPF model does not fit the definition of the marketability discount. A DLOM represents the cost of the loss of resale or transfer flexibility until the restrictions lapse, whereas the VPF model measures the cost of preserving the current market value of the unrestricted asset at the beginning of the restriction period by selling a forward contract. A put-option model is well suited for the DLOM calculation; the VPF model is not. Due to its shortcomings, the VPF model will usually significantly underestimate the DLOM, especially for high-volatility stocks, because it measures the net interest cost of a collar strategy during the restriction period, not the cost of the marketability restrictions.

**References**


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