This article develops a probability weighting methodology for evaluating and supporting an appropriate valuation discount for undivided tenancy in common real estate interests. Traditional cost-of-partition models do not reflect the most probable outcome, but instead the least likely and least profitable outcome for a co-owner seeking a liquidity event. We also develop a supportive methodology for estimates of partition likelihood and discount rates, both of which are also consistent with the issues raised in the *Ludwick*¹ tax court case.

**Overview of Tenancy in Common Valuation**

The standard methodology for appraising fractional interests in real estate is cost-of-partition analysis. This is partly because it is favored by several tax court judges and IRS auditors. However, it is also favored, to the exclusion of other potential analyses, by a large percentage of appraisers hired by tax court petitioners. Such clients benefit when their fractional interests are appraised at a relatively low value, and the cost-of-partition analysis considers the worst possible scenario, leading to a value that reflects a substantial discount. In addition to the cost-of-partition analysis, IRS attorneys sometimes favor conservative assumptions similar to those used by the *Ludwick* court, which were mostly chosen arbitrarily because both appraisers failed to provide support for their chosen methods. While mathematical partition cost analysis appeals to the IRS and tax courts, its implicit assumptions prove difficult to support². This article demonstrates how a more nuanced approach can both (a) rebut the conservatively low assumptions used by the *Ludwick* court, and (b) include alternative scenarios, other than a cost-of-partition analysis, that still address the likelihood of a partition action.

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¹ *Ludwick v. Commissioner*, T.C. Memo. 2010-104 (May 10, 2010).
² The cost-of-partition method, for example, assumes an orderly sale process within a fixed term, reasonable attorney fees, and an absence of property stigma by potential buyers.
Both appraisers in the *Ludwick* case presented comparable sales data relating to fractional interests that supported their selected valuation discounts. The *Ludwick* court, however, criticized the applicability of the comparables presented by both appraisers. In response to these criticisms, I developed a more suitable probability-weighted model that incorporates multiple scenarios similar to the model used by the *Ludwick* court. In short, fractional interests suffer from marketability impairments similar to other extraordinary sale conditions, such as those properties owned by bankruptcy estates and foreclosed properties owned by banks (REO sales). As such, the use of bankruptcy and REO comparable paired-sale data that are similar in property type\(^3\) will likely prove more persuasive than those comparables presented by the appraisers in *Ludwick*.

**Analysis of Ludwick Tax Court Case**

**Overview of Case**

The *Ludwick* case is a published tax court case related to the February 2005 valuation of a 50 percent undivided interest in a Hawaiian vacation home (“Subject Property”). The Subject Property was encumbered by a tenancy in common agreement restricting any partition action; however liquidity was essentially unimpaired as each co-owner retained the right to market the property and retain a pro rata share of the proceeds. The two taxpayers (Petitioners) and their appraiser failed to persuade the U.S. Tax Court to rule in their favor on several key issues. Additionally, the Petitioners failed to address issues that the court found critical. The IRS (Respondent) and its appraiser also failed to persuade the court to accept its analysis. In response, the court effectively developed a weighted average model based upon (a) a sale at fair market value (90 percent probability), and (b) the cost of a partition action (10 percent probability), reflecting each undivided interest’s unimpaired marketability.

\(^3\) Specific sale discounts may be abstracted by comparing bankruptcy and REO-affected comparables to unaffected sale comparables. Paired–sale discounts are calculated by comparing, for example, bankruptcy-affected sale prices to non-affected sales and abstracting adjustments. Similarly paired-sale discounts may be abstracted from REO-affected sales prices.
Use of Undivided Interest Sale Comparables

The taxpayers’ appraiser used a recap of comparable sales data relating to undivided interests in properties, but failed to list the supporting data. Consequently, the appraiser was unable to submit to the court any specific analysis comparing the subject undivided interest to any of the comparables. The Ludwick court criticized the taxpayer’s appraiser for omitting specific details of the comparables, but not necessarily the use of such comparables. However, the court did criticize the IRS appraiser’s use of such comparables and related explanations.

Since the Ludwick court criticized the applicability of the comparables presented by both appraisers, the use of bankruptcy and REO comparable data that are similar in property type may prove more persuasive as proxy data. This proxy data is typically more available in the local market compared to fractional interest sale data.

In addition to using proxy data, it is important to use more than one example. In the Weinberg case, the Tax Court favored the selection of multiple comparables over the opposing expert’s use of only one comparable for the minority partnership interest valuation at issue.

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4 In the instant case, the court criticized the taxpayer’s appraisal report for using summary statistics relating to transaction that were unavailable. The court also criticized the statistical presentation for omitting any statistics relating to variability (such as standard deviation) and “specifics.” In Northern Trust Co., 87 T.C. 324.5 (1986), the court disregarded an appraiser’s analysis when he based the fractional interest discount exclusively on the average discount of a study. The court found that “The valuation of a closely held corporation...must take into account all of the relevant facts and circumstances of the particular corporations under scrutiny.” In Ludwick, the Tax Court disregarded the appraiser’s analysis because it lacked specific analysis of the comparables.

Probability of Partition Action

The *Ludwick* court considered only two alternatives: (a) the Subject Property would be sold at market value (net of sale costs) after one year, and (b) the Subject Property would be sold as part of a two-year partition action. This simple model fails to consider the most likely scenario wherein the non-selling owner, who has more negotiation leverage as the most likely buyer, or another buyer, demands and negotiates a discount similar to the discount resulting from the potential partition action. The *Ludwick* court’s assumption of a 90 percent probability that a buyer would pay the full pro-rata fair market value is unsupported and inconsistent with its model. Even if the non-selling co-owner is financially able and willing to buy the offered 50 percent interest, he or she would not need to pay the brokerage costs or wait one year. The non-selling owner, moreover, remains uniquely motivated to buy the offered undivided interest, as he or she would then enjoy a value enhancement of his or her previously owned undivided interest. In contrast, a third party would be unwilling to pay pro rata for the Subject Property, because it would then own an investment with fewer benefits and more impairments than an alternative property.

Regarding the *Ludwick* case, Exhibit 1 is a summary of several relevant factors that impact the likelihood that a potential buyer of the Subject Property, in my opinion, would require a partition action to achieve a liquidity event. I included it herein as an example for an appraiser to support his or her opinions regarding the likelihood of a partition action. The appraisers’ opinion of such a likelihood was a critical issue in the instant case. The factors are rated from 1 to 10, which represent the likelihood of a partition action occurring when that particular factor exists. With respect to the *Ludwick* case, several of the relevant factors are rated a 5 or 8. This indicates that there is a moderate probability, for a potential buyer of the 50 percent TIC interest, that a partition action would be required. In the end, the court settled on a 10 percent probability of a partition action, mostly because neither party addressed the issue in its briefings to the court.
## EXHIBIT 1

### FACTORS AFFECTING LIKELIHOOD OF REQUIREMENT OF PARTITION ACTION FOR LUDWICK CASE FACT SET

<table>
<thead>
<tr>
<th>Factor</th>
<th>Relevance for Higher Likelihood of Required Partition Action (Scale 1 to 10, 10=maximum.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residence that co-owners use personally</td>
<td>8 (owner-users are more likely to resist partition action.)</td>
</tr>
<tr>
<td>Existing loan financing</td>
<td>1 (no loan, less risk to buyer)</td>
</tr>
<tr>
<td>Number of co-owners</td>
<td>8 (2 total, fewer co-owners make partition action more likely)</td>
</tr>
<tr>
<td>Concentration of ownership</td>
<td>5 (50% max for co-owner, no majority owner)</td>
</tr>
<tr>
<td>Exposure time for Property</td>
<td>5 (1 year marketing time per respondent)</td>
</tr>
<tr>
<td>Fee ownership of Property</td>
<td>1 (Property is fee simple, less risk than leasehold, interest subject to ground lease)</td>
</tr>
<tr>
<td>Property Age and Condition: Unknown</td>
<td></td>
</tr>
<tr>
<td>Consistency of Distributions</td>
<td>10 (Property is unleased, more opportunity cost to buyer)</td>
</tr>
<tr>
<td>TIC Agreement with Partition Action Prohibition</td>
<td>3 (Property is encumbered by a TIC agreement, but each co-owner has a right to market the property)</td>
</tr>
</tbody>
</table>

### Cost of Partition Method in Ludwick Case

The *Ludwick* court questioned the parties’ appraisers regarding the rationale underlying a lack of marketability discount. It agreed such a discount was warranted, although no mention was made of a discount for lack of control. The *Ludwick* court concluded that a buyer of an undivided interest “could not demand a discount greater than (a) the discount reflecting the cost and likelihood of partition and (b) the discount representing a marketability risk.”⁶ The *Ludwick* court, however, did not incorporate marketability risk into its financial model. The court also noted that the Petitioners’ appraiser admitted that a partition action was unlikely between the co-owners.

Since the *Ludwick* court criticized the applicability of the comparables presented by both appraisers, the use of bankruptcy and REO comparable data that are similar in

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⁶ Ludwick v. Commissioner, T.C. Memo. 2010-104, 4-12.
property type may prove more persuasive as proxy data. This proxy data is typically more available in the local market, compared to fractional interest sale data.

In its calculations, the *Ludwick* court found that a hypothetical partition action would take two years. The cost of litigation would be one percent of the property value (or $72,500), and the *Costs of Sale* would total six percent. These expected value figures appear reasonable, but I believe the likelihood of a court action exceeding these figures is much greater than the likelihood of the actual costs falling below the estimates. Lacking any supportable analysis from the taxpayer’s appraiser, the *Ludwick* court used, in its valuation model, a 10 percent “rate of return” or discount rate, as suggested by the IRS’s appraiser.

**Discount Rate Analysis for Partition Cost Model**

**Overview**

For most cost-of-partition cost model analysis, a higher discount rate than that used by the *Ludwick* court is supportable by two methods: the build-up method and institutional investor surveys.

**Discount Rate via Build-Up Method**

Using the build-up method, I started with a base discount rate of 9 percent, which reflects the cash flow discount rate for residential, institutional property investment. The second layer relates to the incremental risk of an extended partition action (exceeding two years), which I estimated at 2.5 percent. The third layer reflects the entrepreneurial reward required to administer the adversarial litigation and is estimated at 2.5 percent. The fourth layer relates to property specific risks; this premium also reflects the Subject Property’s lack of institutional appeal, relative to Class A and B properties referenced in the investor surveys. As the Subject Property is very large and

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7Applicable real estate investment surveys include Price Waterhouse Coopers (pwc.com) and realtyrate.com.
pays no distributions, I added an additional one percent for this layer. The indicated discount rate is calculated as follows:

<table>
<thead>
<tr>
<th>Description</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Real Estate Discount Rate – Institutional Property</td>
<td>9.0%</td>
</tr>
<tr>
<td>Add: Risk of Partition Action Term Premium(^8)</td>
<td>2.5%</td>
</tr>
<tr>
<td>Add: Entrepreneurial/Admin. Premium(^9)</td>
<td>2.5%</td>
</tr>
<tr>
<td>Add: Subject Property Specific Premium(^10)</td>
<td>1.0%</td>
</tr>
<tr>
<td>Total Discount Rate Via Build-Up Method</td>
<td>15.0%</td>
</tr>
</tbody>
</table>

TO BE CONTINUED

About the Author

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\(^8\) This premium reflects the incremental risk associated with the high probability of a partition action requiring more than two years as opposed to less than two years.

\(^9\) This premium reflects the incremental risk and administrative effort required to manage a lawsuit as a litigant.

\(^10\) This premium reflects the incremental risk associated with the Subject Property, such as secondary location, atypical improvements appealing to a limited buyer pool, or local real estate market sluggishness.